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Ion Slip Effect on the Flow Due to a Rotating Disk With Heat Transfer

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Abstract

The steady hydromagnetic flow due to a rotating disk is studied with heat transfer considering the ion slip. The governing equations are solved numerically using finite differences. The results show that the inclusion of the ion slip has important effects on the velocity distribution as well as the heat transfer.

Keywords: Rotating disk flow; Hydromagnetic flow; Heat transfer; Numerical solution; Finite differences

1. Introduction

The hydrodynamic flow due to an infinite rotating disk was first introduced by von Karman (von Karman, 1921). He formulated the problem in the steady state and used similarity transformations to reduce the governing partial differential equations to ordinary differential equations. Asymptotic solutions were obtained for the reduced system of ordinary differential equations (Cochran, 1934). The extension of the steady hydrodynamic problem to the transient state was done by Benton (Benton, 1966). The influence of an external uniform magnetic field on the flow due to a rotating disk was studied (El-Mistikawy and Attia, 1990; El-Mistikawy, 1991) without considering the Hall effect. Aboul-Hassan and Attia (Aboul-Hassan and Attia, 1997) studied the steady hydromagnetic problem taking the Hall effect into consideration. The problem of heat transfer from a rotating disk at a constant temperature was first considered by Millsaps and Pohlhausen (Millsaps, and Pohlhausen, 1952) for a variety of Prandtl numbers in the steady state. Sparrow and Gregg (Sparrow, and Gregg, 1960) studied the steady state heat transfer from a rotating disk maintained at a constant temperature to fluids at any Prandtl number. Later Attia (Attia, 2002) extended the problem discussed in (Millsaps and Pohlhausen, 1952; Sparrow and Gregg, 1960) to the unsteady state in the presence of an applied uniform magnetic field.

The effect of uniform suction or injection through a rotating porous disk on the steady hydrodynamic flow induced by the disk was investigated (Stuart, 1954; Kuiken, 1971; Ockendon, 1972). Later Attia extended the problem to the case of an unsteady hydromagnetic flow in the presence of an external uniform magnetic field without considering the Hall effect (Attia, 1998). The effect of uniform suction or injection on the flow of a conducting fluid due to a rotating disk was studied (Attia and Aboul-Hassan, 2001) considering the Hall current with the neglect of the ion slip.

In the present work the steady hydromagnetic flow of a viscous, incompressible, and electrically conducting fluid due to the uniform rotation of an infinite, non-conducting, disk in an axial uniform steady magnetic field is studied considering the ion slip with heat transfer. The governing non-linear differential

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equations are solved numerically using the finite difference approximations. Some interesting effects for the Hall current, and the ion slip are reported.

2. Basic equations

The disk is assumed to be insulating and rotating in the z=0 plane about the z-axis with a uniform angular velocity ω. The fluid is assumed to be incompressible and has density ρ, kinematical viscosity υ, and electrical conductivity σ. An external uniform magnetic field is applied in the z-direction and has a constant flux density B_o. The magnetic Reynolds number is assumed to be very small, so that the induced magnetic field is negligible. The electron-atom collision frequency is assumed to be relatively high, so that the Hall effect and the ion slip can not be neglected (Sutton and Sherman, 1965). Due to the axial symmetry of the problem about the z-axis, the cylindrical coordinates (r,φ,z) are used. For the sake of definiteness, the disk is taken to be rotating in the positive φ direction. Due to the symmetry about the z=0 plane, it is sufficient to consider the problem in the upper half space only.

The fluid motion is governed by the continuity equation, the Navier-Stokes equation, and the generalized Ohm's law (Sutton and Sherman, 1965) which are respectively given by

$$\vec{\nabla}.\vec{V}=0 \quad (1)$$

$$\rho(\vec{V}.\vec{\nabla})\vec{V}=-\vec{\nabla}p+\mu\nabla^2\vec{V}+\vec{J}x\vec{B}_o \quad (2)$$

$$\vec{J}=\sigma\left[\vec{E}+\vec{V}x\vec{B}_o-\beta(\vec{J}x\vec{B}_o)+\frac{\beta Bi}{B_o}(\vec{J}x\vec{B}_o)x\vec{B}_o\right] \quad (3)$$

where $\vec{E}$ is the electric field which results from charge separation and is in the z direction which implies that it has no effect on Lorentz force term $\vec{J}\wedge\vec{B}_o$ and consequently on the equations of motion. The last term in the last equation expresses the ion slip effect, where β=1/nq is the Hall factor, *n* is the electron concentration per unit volume, -q is the charge of the electron, and Bi is the ion slip parameter (Sutton and Sherman, 1965). In cylindrical coordinates, the velocity vector is written as

$$\vec{V}=u\vec{e}_r+v\vec{e}_\varphi+w\vec{e}_z$$

Expressing the rest of vectors in Eq. (3) in cylindrical coordinates and solving for the unknown vector $\vec{J}$ and, in turn, substituting in Eqs. (1) and (2), the set of equations of steady motion takes the form

$$u_{,r}+\frac{u}{r}+w_{,z}=0 \quad (4)$$

$$\rho(uu_{,r}+wu_{,z}-\frac{v^2}{r})+\frac{\sigma B_o^2}{(\alpha^2+B_e^2)}(\alpha u-Bev)+p_{,r}=\mu(u_{,rr}+\frac{u_{,r}}{r}-\frac{u}{r^2}+u_{,zz}) \quad (5)$$

$$\rho(uv_{,r}+wv_{,z}+\frac{uv}{r})+\frac{\sigma B_o^2}{(\alpha^2+Be^2)}(\alpha v+Beu)=\mu(v_{,rr}+\frac{v_{,r}}{r}-\frac{v}{r^2}+v_{,zz}) \quad (6)$$

$$\rho(uw_{,r}+ww_{,z})+p_{,z}=\mu(w_{,rr}+\frac{w_{,r}}{r}+w_{,zz}) \quad (7)$$

where Be (=σβB_o) is the Hall parameter which can take positive or negative values and α=1+ BeBi. The boundary conditions are given as

$$z=0, u=0, v=r\omega, w=0 \quad (8a)$$

$$z\to\infty, u\to 0, v\to 0, p\to p_\infty \quad (8b)$$

Equation (8a) indicates the no-slip conditions of viscous flow applied at the surface of the disk. Far from the surface of the disk, all fluid velocities must vanish aside the induced axial component as indicated in Eq. (8b). We introduce von Karman transformations [1],

$$u=r\omega F, v=r\omega G, w=\sqrt{\omega\nu}H, z=\sqrt{\nu/\omega}\zeta, p-p_\infty=-\rho\nu\omega P$$

where ζ is a non-dimensional distance measured along the axis of rotation, F, G, H and P are non-dimensional functions of the modified vertical coordinate ζ. We define the magnetic interaction number γ by $\gamma=\sigma B_o^2/\rho\omega$ which represents the ratio between the magnetic force to the fluid inertia force. With these definitions, Eqs. (4)-(8) take the form

$$\frac{dH}{d\zeta}+2F=0 \quad (9)$$

$$\frac{d^2F}{d\zeta^2}-H\frac{dF}{d\zeta}-F^2+G^2-\frac{\gamma}{(\alpha^2+Be^2)}(\alpha F-BeG)=0 \quad (10)$$

$$\frac{d^2G}{d\zeta^2} - H\frac{dG}{d\zeta} - 2FG - \frac{\gamma}{(\alpha^2 + Be^2)}(\alpha G + BeF) = 0 \tag{11}$$

$$\frac{d^2H}{d\zeta^2} - H\frac{dH}{d\zeta} - \frac{dP}{d\zeta} = 0 \tag{12}$$

$$\zeta = 0, F = 0, G = 1, H = 0, \tag{13a}$$

$$\zeta \to \infty, F \to 0, G \to 0, P \to 0, \tag{13b}$$

The above system of Eqs. (9)~(11) with the prescribed boundary conditions given by Eq. (13) are sufficient to solve for the three components of the flow velocity. Equation (12) can be used to solve for the pressure distribution if required.

Due to the difference in temperature between the wall and the ambient fluid heat transfer takes place. The energy equation, by neglecting the dissipations, takes the form (Millsaps and Pohlhausen, 1952; Sparrow and Gregg, 1960),

$$\rho c_p\left(u\frac{\partial T}{\partial r} + w\frac{\partial T}{\partial z}\right) = k\left(\frac{\partial^2 T}{\partial z^2} + \frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r}\right) \tag{14}$$

where T is the temperature of the fluid, c_p is the specific heat at constant pressure of the fluid, and k is the thermal conductivity of the fluid. The boundary conditions for the energy problem are that the temperature by continuity considerations, equals T_w at the surface of the disk. At large distances from the disk, $T \to T_\infty$ where T_∞ is the temperature of the ambient fluid. In terms of the non-dimensional variable $\theta=(T-T_\infty)/(T_w-T_\infty)$ and using Von Karman transformations, Eq. (14) takes the form

$$\frac{1}{\Pr}\frac{d^2\theta}{d\zeta^2} - H\frac{d\theta}{d\zeta} = 0 \tag{15}$$

where Pr is the Prandtl number given by, $Pr=c_p\mu/k$. The boundary conditions for the energy problem, in terms of θ and Von Karman transformations, are expressed as

$$\theta(0) = 1, \zeta \to \infty : \theta \to 0. \tag{16}$$

It should be pointed out that the similarity aspects of the transformation are linked to the supposition that the velocity and temperature do not change shape at different values of r. Also, the idea of angular symmetry has been invoked; i.e. $\partial/\partial\varphi = 0$.

The heat transfer from the disk surface to the fluid is computed by application of Fourier's law

$$Q = -k\left(\frac{dT}{dz}\right)_w \tag{17}$$

Introducing the transformed variables, the expression for q becomes

$$Q = -k(T_w - T_\infty)\sqrt{\frac{\omega}{\nu}}\frac{d\theta(0)}{d\zeta} \tag{18}$$

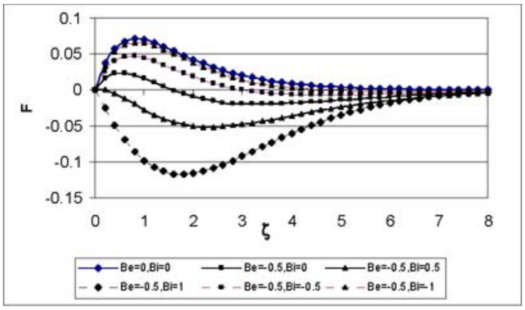
By rephrasing the heat transfer results in terms of a Nusselt number defined as, $N_u = Q\sqrt{\omega/\nu}\,/\,k(T_w - T_\infty)$ the last equation becomes

$$N_u = -\frac{d\theta(0)}{d\zeta} \tag{19}$$

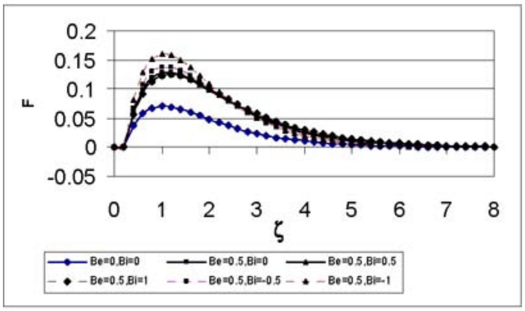
The system of non-linear ordinary differential Eqs. (9)~(11) and (15) is solved under the conditions given by Eqs. (13) and (16) for the three components of the flow velocity and temperature distribution, using the Crank-Nicolson method (Ames, 1977). The resulting system of difference equations has to be solved in the infinite domain $0<\zeta<\infty$. A finite domain in the ζ-direction can be used instead with ζ chosen large enough to ensure that the solutions are not affected by imposing the asymptotic conditions at a finite distance. The independence of the results from the length of the finite domain and the grid density was ensured and successfully checked by various trial and error numerical experimentations. Computations are carried out for ζ_∞=12 and it is found that a value of ζ_∞=10 is adequate for the ranges of the parameters studied here. It should be pointed that the steady state solutions reported by Attia and Aboul-Hassan (Attia and Aboul-Hassan, 2001) are reproduced by setting Bi=0 in the present results. These comparisons lend confidence in the correctness of the solutions presented in this paper.

3. Results and discussion

Figures 1(a) and 1(b) present the profile of the radial velocity component F for various values of the ion slip parameter Bi and for Be≤0 and Be≥0, respectively. In these figures γ=1. Figure 1 shows that for Be= -0.5 and Bi=0, the radial flow reverses its di-rection at a certain distance from the disk. Increasing Bi, for

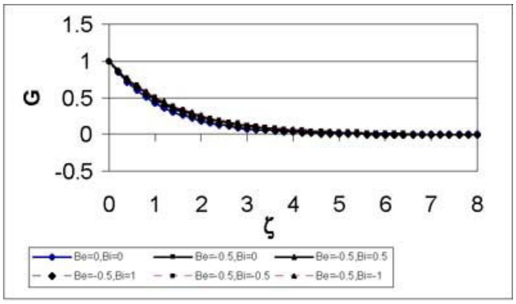


(a) Be<0

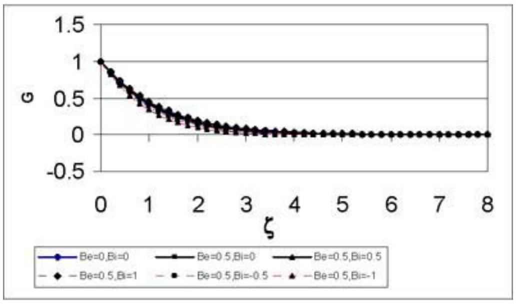


(b) Be>0

Fig. 1. Steady state profile of the radial velocity component *F* for various values of the Hall parameter *Be* and the ion slip parameter *Bi*.

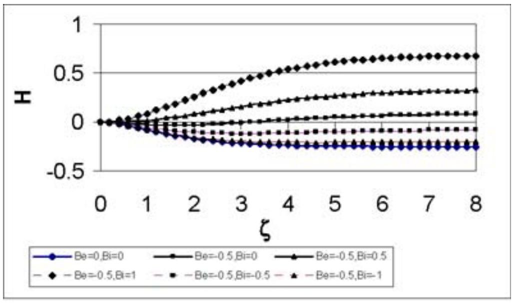


(a) Be<0

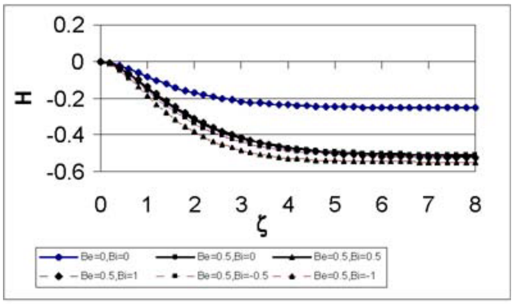


(b) Be>0

Fig. 2. Steady state profile of the azimuthal velocity component *G* for various values of the Hall parameter *Be* and the ion slip parameter *Bi*.



(a) Be<0



(b) Be>0

Fig. 3 Steady state profile of the vertical velocity component *H* for various values of the Hall parameter *Be* and the ion slip parameter *Bi*.

Be<0, leads to a negative F for all ζ as a result of increasing the effective conductivity $(=\gamma/(\alpha^2+Be^2))$ which increases the damping force on F. Figure 1(a) indicates also that for BeBi>0, increasing the magnitude of Bi increases F due to the decrease in the effective conductivity which decreases the damping force on F. Large values of Bi reduce the effective conductivity more which corresponds to the hydrodynamic case. Figure 1(b) indicates that when Be>0 and Be>0, increasing Bi decreases F for some ζ. This may be attributed to the fact that in the magnetic force term in Eq. (7), the effect of Bi on the numerator is stronger than its effect on the denominator which increases the damping force on F and consequently decreases F for some ζ. Also, for Bi<0, increasing the magnitude of Bi increases F for small ζ and then decreases it for larger ζ. This accounts for a crossover in the F-ζ chart with Bi. It is of interest to detect that the variation of F with Bi depends on ζ.

Figures 2(a) and 2(b) present the profile of the azimuthal velocity component G for various values of the ion slip parameter Bi and for Be≤0 and Be≥0, respectively. In both figures, γ=1. As shown in Fig. 2(a), small negative values of Be increases G as a result

Table 1. Variation of the steady state value of N_u with Be, Bi and Pr.

N_u	Pr=0.7	Pr=10
Be=0, Bi=0	0.2232	0.881
Be=-0.5, Bi=0	0.1692	0.712
Be=-0.5, Bi=0.5	0.1318	0.581
Be=-0.5, Bi=1	0.0819	0.394
Be=-0.5, Bi=-0.5	0.1963	0.799
Be=-0.5, Bi=-1	0.2162	0.859
Be=0.5, Bi=0	0.2777	1.058
Be=0.5, Bi=0.5	0.2758	1.043
Be=0.5, Bi=1	0.2232	1.037
Be=0.5, Bi=-0.5	0.1692	1.092
Be=0.5, Bi=-1	0.1318	1.161

of decreasing the magnetic damping. Increasing Bi, with Be<0, decreases G, due to the increase in the effective conductivity. Figure 2(a) shows also that for negative values of Bi, increasing the magnitude of Bi increases G due to the decrease in the damping force on G. Figure 2(b) describes the same findings. For BeBi>0, in-creasing Bi increases G, while for BeBi<0, increasing the magnitude of Bi decreases G.

Figures 3(a) and 3(b) present the profile of the axial velocity component H for various values of the ion slip parameter Bi and Be≤0 and Be≥0, respectively. In these figures γ=1. As shown in Fig. 3(a), for Be=-0.5 and Bi=0, the axial velocity H reverses its direction at a certain ζ. Increasing Bi increases H, as a result of decreasing F, and reverses its direction for all ζ. Figure 3(b) shows that for Be>0, increasing the magnitude of Bi, in general, decreases H as a result of increasing F. It is also shown in Fig. 3(b) that the axial flow is always towards the disk for all values of Bi.

Table 1 present the variation of the Nusselt number N_u with the Hall parameter Be, and the ion slip parameter Bi for Pr=0.7 and 10. In these tables γ=1. It is clear that For Pr=10 and Be<0, increasing Bi increase Nu. For Pr=0.7 and Be<0, increasing Bi decreases Nu, but increasing Bi more increases Nu. For Be>0, increasing the magnitude of Bi increases Nu for all Pr. It is seen in Table 1 that increasing Pr increases N_u. The influence of Be and Bi on N_u is more pronounced for higher Pr than smaller values of Pr. It is seen also that the effect of the ion slip on N_u becomes more apparent for Be<0 than for Be>0 for all values of Pr.

4. Conclusion

The steady MHD flow due to an infinite rotating disk was studied with heat transfer considering the Hall effect and the ion slip. The inclusion of the Hall effect and the ion slip reveals some interesting phenomena and it is found that the signs of the Hall and ion slip parameters are important. It was found that both the radial and vertical velocity components reverse direction for certain values of the magnetic field, the Hall and the ion slip parameters. The variation of the velocity components with the ion slip depends on ζ especially for positive values of the ion slip parameter. Also, the effect of the ion slip parameter is more apparent for positive values of the Hall parameter than negative values. The heat transfer at the surface of the disk is found to depend on the magnitude and the sign of the Hall and ion slip parameters. The dependence of the heat transfer on the Hall and ion slip parameters was found to depend on the Prandtl number and becomes more clear for higher values of the Prandtl number. Also, the variation of the heat transfer with the ion slip was shown to depend upon the sign of the Hall parameter.

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